DISTRIBUTION DERIVATIVE METHOD FOR GENERALISED SINUSOID WITH COMPLEX AMPLITUDE MODULATION

Sašo Muševič
Music Technology Group,
Universitat Pompeu Fabra
Barcelona, Spain
musevic@gmail.com

Jordi Bonada
Music Technology Group,
Universitat Pompeu Fabra
Barcelona, Spain
jordi.bonada@upf.edu

ABSTRACT

The most common sinusoidal models for non-stationary analysis represent either complex amplitude modulated exponentials with exponential damping (cPACED) or log-amplitude/frequency modulated exponentials (generalised sinusoids), by far the most commonly used modulation function being polynomials for both signal families. Attempts to tackle a *hybrid* sinusoidal model, i.e. a generalised sinusoid with complex amplitude modulation were relying on approximations and iterative improvement due to absence of a tractable analytical expression for their Fourier Transform. In this work a simple, direct solution for the aforementioned model is presented.

1. INTRODUCTION

Sinusoidal analysis algorithms' vast area of application, ranging from sound and music analysis [1, 2], medical data analysis [3] and imaging [4], Doppler radar [5] and sonar applications, seismic signal analysis, hydrogen atom spectrum analysis [6], laser technology [7] and financial data analysis [8] had fuelled the research field for decades. Rapidly modulated sinusoids found in most of the aforementioned applications have sparked the interest in non-stationary sinusoidal analysis.

Recent development in this area, have provided efficient and accurate methods for either cPACED or generalised sinusoid model. A somehow hybrid model was attempted with real polynomial amplitude amplitude and frequency modulation, however to authors' knowledge only approximate, iterative-improvement type algorithm were developed to date [9, 10].

A very old idea of TF energy *reassignment* [11] has been a focus of much research lately [12, 13, 14, 15, 16]. The vast variety of modifications of the original reassignment, be it merely a generalisation for higher modulations [17] or redefined for an entirely different model [18], has called for a more general name for this family of algorithms. Recently a *reallocation* [19] of TF energy was proposed and will be used in this work.

The paper is organised as follows: in 2 the hybrid signal model is outlined, while section 3 derives the non-linear multivariate polynomial system and its solution, inspired by the distribution derivative method. Section 4 compares the accuracy and computational complexity of the proposed algorithm to the high-resolution method based on rotational invariance.

2. HYBRID SIGNAL MODEL

The hybrid sinusoidal model will be defined as follows:

$$s(t) = a(t)e^{r(t)}, \tag{1}$$

$$a(t) = \sum_{k=0} a_k m_k(t), a_k \in \mathbb{C} \tag{2}$$

$$r(t) = \sum_{l=0} r_l n_l(t), r_l \in \mathbb{C} \tag{3}$$

where m_k, n_k are the complex amplitude and log-AM/FM model functions respectively. To accommodate for the static amplitude and phase, $m_0 = n_0 = 1$ is assumed. A most common, but by no means mandatory selection for the model functions are polynomials: $m_k = n_k = t^k$.

The above model 1 is ambiguous with respect to parameters r_0 and a_0. To show this, the following derivation is considered:

$$s(t) = a(t)e^{r(t)} = a_0\tilde{a}(t)e^{r_0+\tilde{r}(t)} \tag{4}$$

$$= \tilde{a}(t)\exp(\overbrace{\log(|a_0|) + j\angle(a_0) + r_0}^{\tilde{r}_0} + \tilde{r}(t)), \tag{5}$$

Clearly a_0 and r_0 are in fact the same parameter in either Cartesian or polar coordinates. The decision seems irrelevant, however as will be shown in section 3, using the Cartesian form would result in a rank-deficient system, therefore the model will be constrained to $a_0 = 1$.

It is important to note that since modulation functions are complex they both contribute to overall AM/FM. If the same model functions are used ($m_k = n_k$) that can lead to some ambiguity, especially when the energy of m_k, n_k declines fast with k. Such ambiguity can be demonstrated when using polynomials for the modulation functions $m_k = n_k = t^k$:

$$a(t)e^{r(t)} = \exp(\log(a(t)) + r(t)) \tag{6}$$

$$\approx \exp(a_1 t + (2a_2 - {a_1}^2)t^2 + r(t)), \tag{7}$$

using the 2^{nd} degree truncated Taylor expansion. It is expected that an estimator for the model in 1 could be inaccurate when separate parameter estimates are considered, but generally much more flexible due to twin AM/FM functions. In practice however one is mostly concerned with algorithm's overall ability to fit to the signal under investigation, rather then individual per-parameter accuracy. It would be feasible to devise a disambiguation procedure but this is considered to be outside the scope of this document.

An example is shown in figure 1, where 2 hybrid model sinusoids with significantly different a_1, r_1 reach a signal-to-residual ratio (SRR) of $24dB$. To reach higher SRR the parameters would

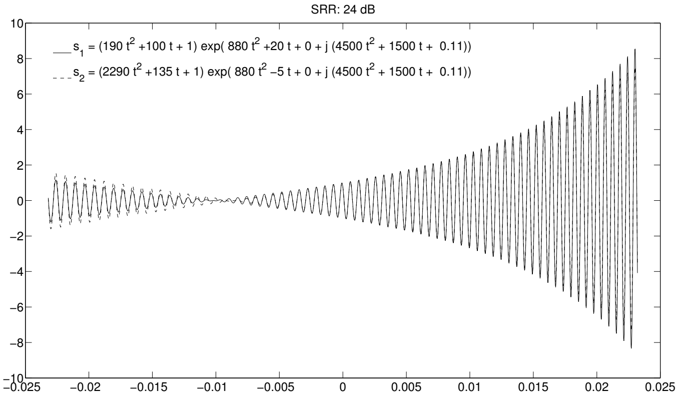

Figure 1: Two sinusoids with significantly different parameters obtain a similar SRR of 24dB.

have to eventually match exactly, however in noisy conditions a relatively high SRR is achievable with substantial error in per-parameter estimates. From figure 1 is also evident that the proposed model includes sinusoids with *negative* amplitude, suggesting good coding abilities for sinusoid pairs with close frequencies (i.e.: beating partials). The negative amplitude can occur when $\Im[a(t)] = 0$, since the overall amplitude corresponds to: $\sqrt{a(t)\bar{a}(t)}e^{\Re[r(t)]}$. The notion of negative amplitude is purely artificial ($\sqrt{a(t)\bar{a}(t)}e^{\Re[r(t)]}$ cannot be negative), as it does not have a natural physical meaning, however it comes handy as a mathematical generalisation. If negative amplitude is not allowed and $\Im[a(t)] = 0$, the derivative of $\sqrt{a(t)\bar{a}(t)}e^{\Re[r(t)]} = |a(t)|e^{\Re[r(t)]}$ is not continuous for all roots of $a(t)$ and thus the model function could not be considered a holomorphic function. In such cases the absolute value can be easily dropped and negative amplitude introduced, leading to a mathematically sound model in the context of holomorphic functions.

3. NON-LINEAR MULTIVARIATE POLYNOMIAL SYSTEM OF EQUATIONS

The non-linear system can be derived by considering the signal time derivative manipulated in the following way:

$$s'(t) = a'(t)e^{r(t)} + r'(t)a(t)e^{r(t)}, \Rightarrow \tag{8}$$

$$a(t)s'(t) = a'(t)s(t) + a(t)r'(t)s(t). \tag{9}$$

The last row can be rewritten in a more verbose form that reveals the non-linearity of the system:

$$m_0 s' + \sum_{k=1}^{K} a_k m_k s' = \sum_{k=1}^{K} a_k m_k' s + \left(m_0 + \sum_{k=1}^{K} a_k m_k\right) \sum_{l=1}^{L} r_l n_l' s, \tag{10}$$

where time variable t was omitted for compactness. The only non-linear terms arise from the last - double sum expression. Multiplying both sides with a window function $w(t)$ and taking a Fourier Transform (FT) at frequency ω yields:

$$S'_{wm_0}(\omega) + \sum_{k=1}^{K} a_k S'_{wm_k}(\omega) =$$
$$\sum_{k=1}^{K} a_k S_{wm_k'}(\omega) + \sum_{l=1}^{L} r_l S_{wm_0 n_l'}(\omega) + \sum_{k=1}^{K}\sum_{l=1}^{L} a_k r_l S_{wm_k n_l'}(\omega), \tag{11}$$

where $S_f(\omega) = \langle s(t)f(t), e^{j\omega t}\rangle$ is the FT of the signal s multiplied by function f, and $S'_g(\omega) = \langle s'(t)g(t), e^{j\omega t}\rangle$ is the FT of the signal derivative multiplied by function g at frequency ω. Note that $n_0' = m_0' = 0$ and thus the sums on the right-hand side start at index 1 rather than 0. Above equation can be viewed as a (non-linear) multivariate polynomial with respect to parameters $a_k : k = 1 \dots K, r_l : l = 1 \dots L$. The expressions S'_f, S_g can be considered constants for any f, g as they can be directly computed from the signal. To calculate S'_f accurately, sample difference in time domain should be avoided [12]. A common approach is the use of distribution derivative rule $\langle x', y\rangle = -\langle x, y'\rangle$ and a real window function w as a part of the kernel y:

$$S'_{gw}(\omega) = \langle s'g, w\psi_\omega\rangle = \langle s', \bar{g}w\psi_\omega\rangle \tag{12}$$

$$= -\langle s, \bar{g}'w\psi_\omega\rangle - \langle s, \bar{g}w'\psi_\omega\rangle + j\omega\langle s, \bar{g}w\psi_\omega\rangle \tag{13}$$

$$= -\langle sg', w\psi_\omega\rangle - \langle sg, w'\psi_\omega\rangle + j\omega\langle sg, w\psi_\omega\rangle \tag{14}$$

$$= -S_{g'w}(\omega) - S_{gw'}(\omega) + j\omega S_{gw}(\omega), \tag{15}$$

where ψ_ω is generally a kernel function with FT centred around frequency ω. For the last equality to hold the kernel is set simply to the Fourier kernel: $\psi_\omega(t) = e^{j\omega t}$. Higher time derivatives can accurately be computed by chaining the above expression. Rearranging the equation and collecting together the model parameters yields:

$$S'_{wm_0}(\omega) = \sum_{k=1}^{K} a_k (S_{wm_k'}(\omega) - S'_{wm_k}(\omega)) +$$
$$\sum_{l=1}^{L} r_l S_{wm_0 n_l'}(\omega) + \sum_{k=1}^{K}\sum_{l=1}^{L} a_k r_l S_{wm_k n_l'}(\omega). \tag{16}$$

Taking the FT at different frequencies close to the peak provides as many equations as necessary. Assuming polynomial modulation functions, the following system can be derived:

$$S'_w(\omega) + \sum_{k=1}^{K} a_k S'_{t^k w}(\omega) = \sum_{k=1}^{K-1} k a_k S_{t^{k-1}w}(\omega)$$
$$+ \sum_{k=1}^{K} a_K \sum_{l=0}^{L-1} (l+1) r_{l+1} S_{t^{k+l}}(\omega) + \sum_{l=0}^{L-1} (l+1) r_{l+1} S_{t^l}(\omega) \tag{17}$$

For a cPACED sinusoids with polynomial amplitude of degree 3 (ie: K=3, L=1) the following case can be deduced:

$$S'_w(\omega) + a_1 S'_{tw}(\omega) + a_2 S'_{t^2 w}(\omega) + a_3 S'_{t^3 w}(\omega) =$$
$$a_1 S_w(\omega) + 2a_2 S_{tw}(\omega) + 3a_3 S_{t^2 w}(\omega) + a_1 r_1 S_{tw}(\omega)$$
$$+ a_2 r_1 S_{t^2 w}(\omega) + a_3 r_1 S_{t^3 w}(\omega) + r_1 S_w(\omega) \tag{18}$$

Grouping the linear and non-linear terms in respect to a_l, r_1:

$$S'_w(\omega) = a_1(S_w(\omega) - S'_{tw}(\omega)) + a_2(2S_{tw}(\omega) - S'_{t^2w}(\omega)) + a_3(3S_{t^2w}(\omega) - S'_{t^3w}(\omega)) + r_1 S_w(\omega) + a_1 r_1 S_{tw}(\omega) + a_2 r_1 S_{t^2w}(\omega) + a_3 r_1 S_{t^3w}(\omega). \quad (19)$$

The distribution derivative rule can be applied to the S' terms:

$$S'_{t^k w}(\omega) = -kS_{t^{k-1}w}(\omega) - S_{t^k w'}(\omega) + j\omega S_{t^k w}(\omega), \quad \text{for } k > 0 \quad (20)$$

$$S'_w(\omega) = -S_{w'}(\omega) + j\omega S_w(\omega), \quad \text{for } k = 0 \quad (21)$$

Following the approach of the distribution derivative method and considering the FT at 4 different frequencies, we obtain the following non-linear multivariate system:

$$Ax = b \quad (22)$$

$$A = \begin{pmatrix} S_w(\omega_1) - S'_{tw}(\omega_1) & \cdots & S_w(\omega_4) - S'_{tw}(\omega_4) \\ 2S_{tw}(\omega_1) - S'_{t^2w}(\omega_1) & \cdots & 2S_{tw}(\omega_4) - S'_{t^2w}(\omega_4) \\ 3S_{t^2w}(\omega_1) - S'_{t^3w}(\omega_1) & \cdots & 3S_{t^2w}(\omega_4) - S'_{t^3w}(\omega_4) \\ S_w(\omega_1) & \cdots & S_w(\omega_4) \\ S_{tw}(\omega_1) & \cdots & S_{tw}(\omega_4) \\ S_{t^2w}(\omega_1) & \cdots & S_{t^2w}(\omega_4) \\ S_{t^3w}(\omega_1) & \cdots & S_{t^3w}(\omega_4) \end{pmatrix}' \quad (23)$$

$$x = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ r_1 \\ r_1 a_1 \\ r_1 a_2 \\ r_1 a_3 \end{pmatrix}, b = \begin{pmatrix} S'_w(\omega_1) \\ S'_w(\omega_2) \\ S'_w(\omega_3) \\ S'_w(\omega_4) \end{pmatrix}.$$

Note that for high parameter values, the frequency spread of the signal might be large - a small number of frequency bins (in the above case 4) might not suffice to cover enough information in the Fourier domain. In such cases more frequency bins can be considered.

4. TESTS AND RESULTS

The proposed method was implemented in a Matlab script for an arbitrary degree of amplitude and exponential complex polynomials. The resulting multivariate non-linear systems were solved by the Matlab function $fsolve$. This function only accepts real variables and coefficients as parameters, although internally can use complex variables and solve the system if the complex equations are split into real and imaginary parts.

A polynomial amplitude of degree 3 was studied and the polynomial denoted as: $[a_3, a_2, a_1, 1] = [p_3 + jq_3, p_2 + jq_2, p_1 + jq_1, 1]$. The test values for p_3, p_2, p_1 were chosen so all the terms of the amplitude polynomial have equal impact on the final value:

$$p_3 \in \left[-\left(\frac{fs}{8T}\right)^3, \left(\frac{fs}{8T}\right)^3\right] \quad (24)$$

$$p_2 \in \left[-\left(\frac{fs}{8T}\right)^2, \left(\frac{fs}{8T}\right)^2\right] \quad (25)$$

$$p_1 \in \left[-\frac{fs}{8T}, \frac{fs}{8T}\right]. \quad (26)$$

The exact same value sets were used for the imaginary part of the polynomial $q(t)$. A Hann window function of length 511 samples was used for pole estimation and Hann window for the complex polynomial coefficients estimation. The damping factor was set to [-150,0,150] and only one frequency of 10000Hz was considered to match tests performed in [12]. r_0 was set to 0, since gain has theoretically no effect when the snr is fixed. For p_1, p_2, p_3, q_1, q_2 and q_3 parameters, only 5 linearly distributed values have been tested in order to keep the computational time reasonable. The comparison to a 3^{th} degree (i.e. 4 poles and amplitudes) simple high-resolution method (HRM) implementation from DESAM Toolbox [20] (section 5.1.2.) without whitening and the cPACED reassignment method (cPACED-RM) [18] was conducted. The signal tested is the real part of the complex cPACED signal, reflecting the real world scenario when analytical signal isn't available.

To measure accuracy, the commonly used Signal-to-Residual-Ratio (SRR) metric was used,

$$SRR = \frac{\langle s, ws \rangle}{\langle s - \hat{s}, w(s - \hat{s}) \rangle}, \quad (27)$$

where $s, \hat{s}$ are the original signal (without noise) and the estimated signal respectively, and w is the Hann window. The Signal-to-Noise-Ratio (SNR) range from -20 to 50dB with steps of 10dB was studied. The total computation times for both methods follow:

cPACED-RM	7 min
cPACED-DDM	300 min
High-resolution	880 min

(28)

Since HRM involves singular value decomposition of correlation matrix of size $N/2 \times N/2$ the computation cost is significantly the highest among the tested methods. cPACED-RM method requires $K - 1$ FFTs for the pole and K DTFTs for the complex polynomial estimates to build a linear system. In contrast, the proposed method requires only K DTFTs to build a non-linear multivariate system. However, solving such system by iterative methods requires a significant computation cost. cPACED-RM method performs much faster since it requires solving a linear system.

The classic Cramer-Rao bounds (CRBs) parameter-by-parameter comparison would total to 10 plots, overcomplicating the results and obscuring the overall accuracy. A more intuitive approach involves only one SRR/SNR plot, although a different upper accuracy bound is required. For each test case the CRBs for each parameter were computed. Denoting a CRB for parameter a_0 as ϵ_{a_0}, the minimum SRR for the specific CRB set can be defined:

$$\min SRR(\hat{s}(a_3 \pm \epsilon_{a_3}, a_2 \pm \epsilon_{a_2}, a_1 \pm \epsilon_{a_1}, r_0 \pm \epsilon_{r_0}, r_1 \pm \epsilon_{r_1}). \quad (29)$$

The mean and variance of the minimum SRR represents a good upper SRR bound. Figure 2 depicts the mean and variance of the upper SRR bound, the proposed method (cPACED-DDM), cPACED-RM and HRM. At low SNR, cPACED-DDM and cPACED-RM perform roughly the same, up to ˜5dB below the upper bound, while cPACED-RM performs ˜3dB better. For mid-high SNRs, all methods perform roughly the same, about ˜5dB below the upper bound. In general cPACED-DDM performs ˜1dB below cPACED-RM.

The main advantage of the proposed algorithm is that it offers the flexibility to analyze generalised sinusoids with complex amplitude of any polynomial degree combination. With the purpose of having an initial exploration of its general performance, we have computed the mean SRR obtained for several combinations of amplitude and exponential complex polynomial degrees. In particular, amplitude polynomial degrees were set to [0,1,2,3] and exponential polynomial degrees to [1,2,3]. Table 1 shows the results obtained. In this experiment we used the same polynomial coefficient value sets as for the previous cPaced case. For each SNR, the results show a general tendency to decrease the SRR as we increase the complexity of the signal by increasing the degrees of the polynomials. Figure 3 compares the mean and variance of the SRR obtained for all degree combinations with the SNR. It shows a general trend of reaching an SRR ˜24dB above the SNR value, although the difference decreases significantly to ˜13dB for SNR=-20dB, and to ˜14dB for SNR=50dB. This seems to indicate that the proposed method reaches a plateau for high SNRs above 50dB.

5. DISCUSSION AND FUTURE WORK

In this work the currently most flexible sinusoidal method for TF energy reassignment analysis has been described. The concept used in the distribution derivative method is used to generate a nonlinear multivariate system of polynomials obtained by the first signal derivative. It is important to note that higher signal derivatives would provide enough equations for a solution to exist, however a significantly more complex system would be obtained. Even if solution could eventually be obtained, it is desirable to avoid higher signal derivatives due to ill conditioning.

The method showed a similar performance than the high resolution and the reassignment methods for the cPACED signal model, however in theory the proposed method is much more flexible since it can be applied to generalised sinusoids with complex amplitude of any polynomial degree combination. The initial exploration of the performance obtained for several degree combinations was promising, although a more in depth evaluation was left for future work. Further, the proposed sinusoidal model seems promising for the analysis of overlapping partials, as the beating function corresponds to real value amplitude/frequency modulated sinusoids - a subfamily of signals described by the proposed model.

On the other hand high-resolution methods' intrinsic frequency resolution of 1 frequency bin [21, 22] for damped sinusoids has not been surpassed, as common window function mainlobe width (several bins) and significant sidelobe amplitude both reduce the frequency resolution.

6. ACKNOWLEDGMENTS

This research is partially funded by the SIGMUS project (TIN2012-36650).

7. REFERENCES

[1] J. Bonada, O. Celma, A. Loscos, J. Ortolà, and X. Serra, "Singing voice synthesis combining excitation plus resonance and sinusoidal plus residual models," La Habana, Cuba, Sep. 2001. [Online]. Available: http://quod.lib.umich.edu/cgi/p/pod/dod-idx?c=icmc;idno=bbp2372.2001.093

[2] X. Serra, "A system for sound Analysis/Transformation/Synthesis based on a deterministic plus stochastic decomposition," Ph.D. dissertation, Stanford University, Stanford, CA, 1989. [Online]. Available: https://ccrma.stanford.edu/files/papers/stanm58.pdf

[3] S. M. I. Daubechies, *Wavelets in Medicine and Biology*. CRC Press, 1996, ch. A nonlinear squeezing of the continuous wavelet transform based on auditory nerve models, pp. 527–546.

[4] M. Pattichis, C. Pattichis, M. Avraam, A. Bovik, and K. Kyriacou, "Am-fm texture segmentation in electron microscopic muscle imaging," *Medical Imaging, IEEE Transactions on*, vol. 19, no. 12, pp. 1253–1257, 2000.

[5] A. Sabatini, "Correlation receivers using laguerre filter banks for modelling narrowband ultrasonic echoes and estimating their time-of-flights," *Ultrasonics, Ferroelectrics and Frequency Control, IEEE Transactions on*, vol. 44, no. 6, pp. 1253–1263, 1997.

[6] T. W. Hänsch, A. L. Schawlow, and G. W. Series, "The spectrum of atomic hydrogen," *Sci. Amer.*, vol. 240, no. 94, pp. 531–544, Mar. 1979.

[7] P. W. Milonni and J. H. Eberly, *Lasers*, ser. Wiley Series in Pure and Applied Optics. New York: Wiley-Interscience, 1988.

[8] D. Filipovic, "Exponential-polynomial families and the term structure of interest rates," *Bernoulli*, vol. 6, no. 6, pp. 1081–1107, 2000.

[9] B. Friedlander and J. Francos, "Estimation of amplitude and phase of non-stationary signals," in *Signals, Systems and Computers, 1993. 1993 Conference Record of The Twenty-Seventh Asilomar Conference on*, 1993, pp. 431–435 vol.1.

[10] Y. Pantazis, O. Rosec, and Y. Stylianou, "Adaptive am/fm signal decomposition with application to speech analysis," *Audio, Speech, and Language Processing, IEEE Transactions on*, vol. 19, no. 2, pp. 290–300, 2011.

[11] F. Auger and P. Flandrin, "Improving the readability of time-frequency and time-scale representations by the reassignment method," *Signal Processing, IEEE Transactions on*, vol. 43, no. 5, pp. 1068–1089, 1995.

[12] S. Marchand and P. Depalle, "Generalization of the derivative analysis method to Non-Stationary sinusoidal modeling," *Digital Audio Effects, 2008. Proceedings of the 11th Int. Conference on (DAFx-08)*, 2008. [Online]. Available: http://hal.archives-ouvertes.fr/hal-00351950/en/

[13] M. Betser, "Sinusoidal polynomial parameter estimation using the distribution derivative," *Signal Processing, IEEE Transactions on*, vol. 57, no. 12, pp. 4633 –4645, dec. 2009.

[14] S. Muševič and J. Bonada, "Comparison of non-stationary sinsoid estimation methods using reassignment and derivatives," Barcelona, Spain, Jul. 2010. [Online]. Available: http://smcnetwork.org/files/proceedings/2010/14.pdf

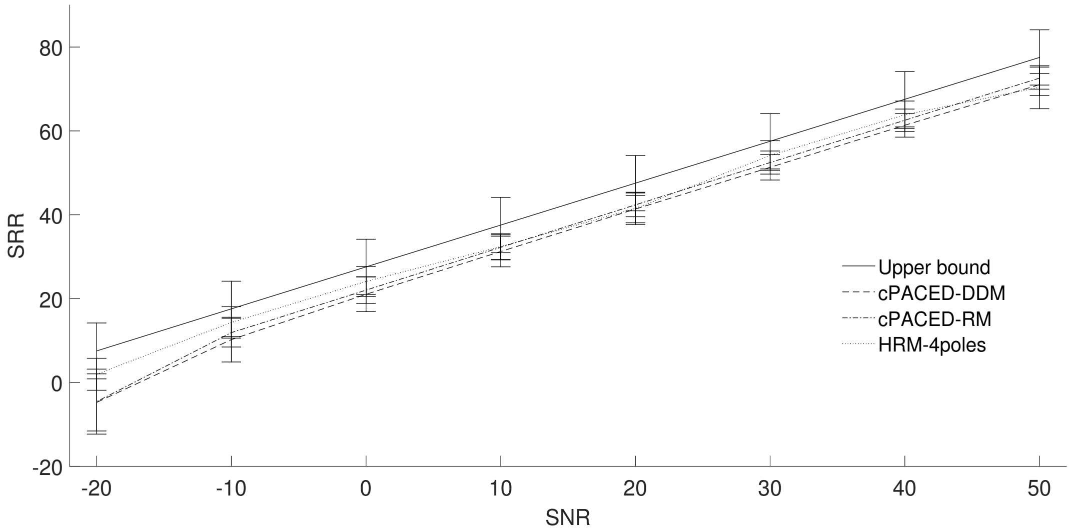

Figure 2: SRR mean and variance for cPaced signals.

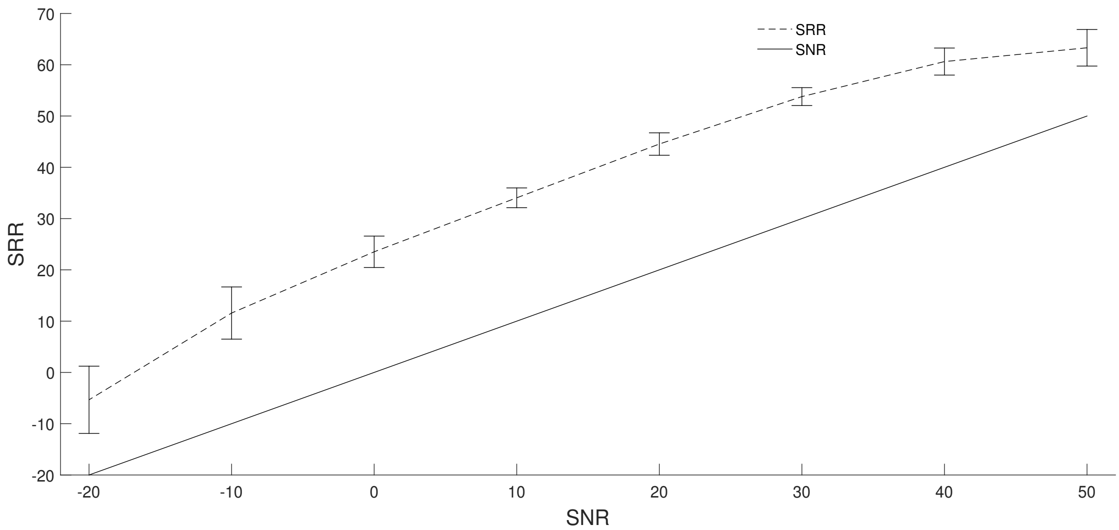

Figure 3: mean and variance of SRR means for all combinations of [0,1,2,3] amplitude polynomial degrees and [1,2,3] exponential polynomial degrees sets. Details are given in Table 1.

SNR	a0r1	a0r2	a0r3	a1r1	a1r2	a1r3	a2r1	a2r2	a2r3	a3r1	a3r2	a3r3
50	72.2	66.7	64.6	62.8	63.9	62.3	64.2	61.2	60.5	62.3	60.8	58.2
40	67.0	63.1	61.7	59.2	61.3	60.0	61.7	59.2	58.7	60.1	58.9	56.6
30	56.6	56.5	54.2	54.3	54.6	53.2	54.5	53.0	52.4	53.3	52.4	50.5
20	50.0	46.4	44.7	44.9	45.2	43.8	44.8	43.6	42.9	44.0	43.0	41.1
10	36.2	35.2	34.7	36.8	34.8	33.6	35.0	33.5	32.8	33.9	32.7	29.4
0	25.9	25.7	24.9	26.4	25.2	23.2	25.2	23.1	22.1	23.8	21.7	15.1
-10	13.7	16.8	14.9	15.4	14.6	10.3	14.7	10.9	8.4	13.3	7.8	-1.8
-20	3.2	1.4	-3.0	-1.5	-2.4	-7.4	-1.6	-6.9	-13.1	-2.5	-10.4	-19.8

Table 1: SRR mean for several polynomial degree combinations. Column numbers in top row indicate amplitude and exponential complex polynomial degrees respectively (e.g. a1r3 means amplitude polynomial degree 1 and exponential polynomial degree 3).

[15] B. Hamilton and P. Depalle, "A unified view of non-stationary sinusoidal parameter estimation methods using signal derivatives," in *Acoustics, Speech and Signal Processing (ICASSP), 2012 IEEE International Conference on*, march 2012, pp. 369 –372.

[16] E. Chassande-Mottin, F. Auger, and P. Flandrin, *Reassignment*. ISTE, 2010, pp. 249–277. [Online]. Available: http://dx.doi.org/10.1002/9780470611203.ch9

[17] M. S. Xue Wen, "Notes on model-based non-stationary sinusoid estimation methods using derivative," in *Digital Audio Effects, 2009. Proceedings of the 12th Int. Conference on (DAFx-09)*, 2009.

[18] J. B. Sašo Muševič, "Derivative analysis of complex polynomial amplitude, complex exponential with exponential damping," in *Acoustics, Speech and Signal Processing (ICASSP), 2013 IEEE International Conference on*, 2013.

[19] I. Daubechies, J. Lu, and H.-T. Wu, "Synchrosqueezed wavelet transforms: An empirical mode decomposition-like tool," *Applied and Computational Harmonic Analysis*, vol. 30, no. 2, pp. 243 – 261, 2011. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S1063520310001016

[20] R. Badeau, B. David, N. Bertin, J. Echeveste, M. Lagrange, O. Derrien, and S. Marchand, "Desam toolbox v1.1." [Online]. Available: http://www.tsi.telecom-paristech.fr/aao/en/2010/03/29/desam-toolbox-2/

[21] B. Porat and B. Friedlander, "On the accuracy of the kumaresan-tufts method for estimating complex damped exponentials," *Acoustics, Speech and Signal Processing, IEEE Transactions on*, vol. 35, no. 2, pp. 231–235, 1987.

[22] Y. Hua and Y. Zhang, "Perturbation analysis of tk method for harmonic retrieval problems," *Acoustics, Speech and Signal Processing, IEEE Transactions on*, vol. 36, no. 2, pp. 228–240, 1988.

[23] Sage mathematical software. [Online]. Available: http://www.sagemath.org

[24] S. Scholler and P. Purwins, "Sparse approximations for drum sound classification," *IEEE Journal of Selected Topics in Signal Processing*, vol. 5, pp. 933 – 940, 2011 2011.

[25] M. Betser, P. Collen, G. Richard, and B. David, "Estimation of frequency for AM/FM models using the phase vocoder framework," *Signal Processing, IEEE Transactions on*, vol. 56, no. 2, pp. 505–517, 2008.

[26] R. Badeau, "Méthodes à haute résolution pour l'estimation et le suivi de sinusoïdes modulées. application aux signaux de musique," Ph.D. dissertation, École Doctorale d'Informatique, Télécommunications et Électronique de Paris, 2005.

[27] M. Bartkowiak, "A complex envelope ssinusoidal model for audio codig," 2007.

[28] M. Christensen and S. van de Par, "Efficient parametric coding of transients," *Audio, Speech, and Language Processing, IEEE Transactions on*, vol. 14, no. 4, pp. 1340 –1351, july 2006.

[29] R. McAulay and T. Quatieri, "Speech analysis/synthesis based on a sinusoidal representation," *Acoustics, Speech and Signal Processing, IEEE Transactions on*, vol. 34, no. 4, pp. 744 – 754, aug 1986.

[30] G. Kafentzis, Y. Pantazis, O. Rosec, and Y. Stylianou, "An extension of the adaptive quasi-harmonic model," in *Acoustics, Speech and Signal Processing (ICASSP), 2012 IEEE International Conference on*, march 2012, pp. 4605 –4608.

[31] K. Hermus, W. Verhelst, P. Lemmerling, P. Wambacq, and S. V. Huffel, "Perceptual audio modeling with exponentially damped sinusoids," *Signal Processing*, vol. 85, no. 1, pp. 163 – 176, 2005. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0165168404002361

[32] J. Jensen, R. Heusdens, and S. Jensen, "A perceptual subspace approach for modeling of speech and audio signals with damped sinusoids," *Speech and Audio Processing, IEEE Transactions on*, vol. 12, no. 2, pp. 121 – 132, march 2004.

[33] J. Nieuwenhuijse, R. Heusens, and E. Deprettere, "Robust exponential modeling of audio signals," in *Acoustics, Speech and Signal Processing, 1998. Proceedings of the 1998 IEEE International Conference on*, vol. 6, may 1998, pp. 3581 – 3584 vol.6.

[34] R. Badeau, B. David, and G. Richard, "High-resolution spectral analysis of mixtures of complex exponentials modulated by polynomials," *Signal Processing, IEEE Transactions on*, vol. 54, no. 4, pp. 1341 – 1350, april 2006.

[35] M. Desainte-Catherine and S. Marchand, "High precision fourier analysis of sounds using signal derivatives," *Audio Engineering Society, 1953 Journal of*, 1998. [Online]. Available: http://citeseer.ist.psu.edu/desainte-catherine98high.html

[36] P. Djuric and S. Kay, "Parameter estimation of chirp signals," *Acoustics, Speech and Signal Processing, IEEE Transactions on*, vol. 38, no. 12, pp. 2118–2126, 1990.